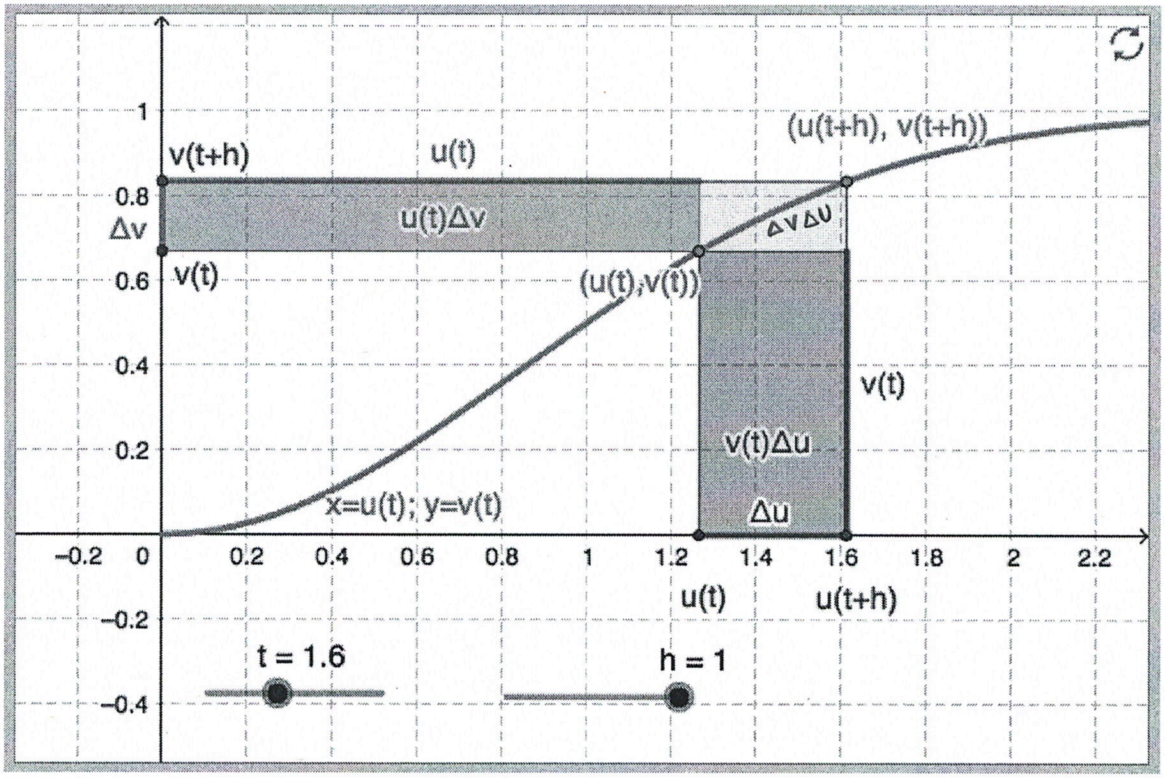
# Product Rule




$\boxed{\text{Prove } (U(t)\cdot V(t))' = U'(t)\,V(t) + U(t)\,V'(t)}$

<u>Notation</u>: $\Delta U = U(t+h) - U(t)$, so $U'(t) = \lim\limits_{h\to 0} \dfrac{\Delta U}{h}$

$\Delta V = V(t+h) - V(t)$, so $V'(t) = \lim\limits_{h\to 0} \dfrac{\Delta V}{h}$

$$(U(t)\cdot V(t))' = \lim_{h\to 0} \frac{U(t+h)V(t+h) - U(t)V(t)}{h}$$

$$= \lim_{h\to 0} \frac{U(t)\Delta V + \Delta U\,V(t) + \Delta U \Delta V}{h}$$ from diagram Area of 3 rectangles

$$= \lim_{h\to 0} \frac{U(t)\Delta V}{h} + \lim_{h\to 0} \frac{\Delta U\,V(t)}{h} + \lim_{h\to 0} \frac{\Delta U \Delta V}{h}$$

$$= U(t)\lim_{h\to 0} \frac{\Delta V}{h} + V(t)\lim_{h\to 0} \frac{\Delta U}{\Delta h} + \lim_{h\to 0} \frac{\Delta U \Delta V}{h}\cdot\frac{h}{h}$$ unusual trick!

$$= U(t)\,V'(t) + V(t)\,U'(t) + \lim_{h\to 0} \frac{\Delta U}{h}\cdot\frac{\Delta V}{h}\cdot h$$ separate into product of limits

$$= \text{''} \qquad \text{''} + \lim_{h\to 0} \frac{\Delta U}{h}\cdot\lim_{h\to 0} \frac{\Delta V}{h}\cdot\lim_{h\to 0} h$$

$$= \text{''} \qquad \text{''} + \cancel{U'(t)\cdot V'(t)\cdot 0} \to 0$$

$$= \boxed{U'(t)V(t) + U(t)V'(t)\ \blacksquare}$$ with terms rearranged